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SHORT NOTE

Polar Mohr constructions for strain analysis in general shear zones

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Abstract—The polar Mohr diagram is a useful tool for strain analysis in general shear zones. Polar Mohr diagrams for general shear zones can be constructed with the following measured data: (a) the stretches of two line markers which were originally perpendicular to each other; (b) the ratio of their stretches and the principal directions; (c) the stretches in the shear direction and in the principal direction; and (d) the stretches in any two directions. With these diagrams, the angle between eigenvectors, v, can be obtained, and the kinematic vorticity number W_k in the shear zone and the ratio of pure shear rate $\dot{\varepsilon}$ to simple shear rate $\dot{\gamma}$ during deformation can be computed with the formulae $W_k = \cos v$ and $W_k = \cos[\cot(2\dot{\varepsilon}/\dot{\gamma})]$. © 1997 Elsevier Science Ltd.

INTRODUCTION

Most natural shear zones are probably formed by general shear which combines pure shear and orthogonal simple shear parallel to the shear zone's boundary (Matthews *et al.*, 1974; De Paor, 1983; Simpson and De Paor, 1993). The ratio of pure shear rate $\dot{\varepsilon}$ to simple shear rate $\dot{\gamma}$ in a general shear zone can be expressed by the kinematic vorticity number W_k (Truesdell, 1954) with the formula $W_k = \cos[\cot(2\dot{\varepsilon}/\dot{\gamma})]$, and the angle v between the two eigenvectors e_1 and e_2 in a deformation field can be used to compute W_k using the formula $W_k = \cos v$ (Bobyarchick, 1986). Thus, $v = 90°$ and $W_k = 0$ stand for pure shear (Fig. 1a); $v = 0°$ and $W_k = 1$ represents simple shear (Fig. 1b); $0° < v < 90°$ and $0 < W_k < 1$ corresponds to general shear (Fig. 1c & d) (Means *et al.*, 1980). The polar Mohr diagram (De Paor, 1983; Simpson and De Paor, 1993), which evolved from the stretch Mohr circle (Choi and Hsü, 1971; De Paor, 1981; Means, 1982, 1983), is a useful tool for determination of the angle v and other strain analyses in a general shear zone. The polar Mohr diagram uses polar co-ordinates to express strain and can be applied to off-axis analyses, making it suitable for both coaxial and non-coaxial deformations. It combines the Mohr space and geographic space, and provides many advantages for strain analyses. Polar Mohr constructions for general shear zones are the key to these analyses and a procedure has been put forward by Simpson and De Paor (1993). Three more practical methods are presented in this paper in order to expand the application of polar Mohr analysis.

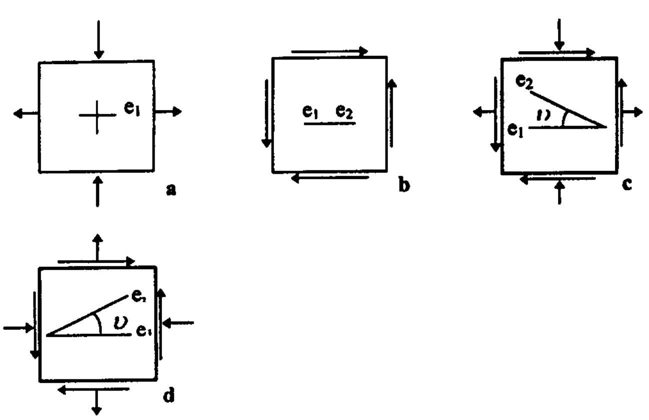

Fig. 1. The relationship between the angle between eigenvectors, v, and deformation histories. (a) Pure shear. (b) Simple shear. (c) General shear which thins the shear zone. (d) General shear which thickens the shear zone.

CONSTRUCTION OF POLAR MOHR DIAGRAMS FOR GENERAL SHEAR ZONES

Simpson and De Paor (1993) have presented a basic method for construction of a polar Mohr diagram using the measured stretch ξ_1 along the shear direction, and the stretch and angular shear (S_0, ψ_0) of a marker originally perpendicular to the shear zone's boundary (Fig. 2a). Zheng and Wang (1995) constructed a polar Mohr diagram using this method for the shear zone in the Yagan metamorphic core complex in north China, and the results from it coincided with the real measured data. The markers employed in this method, however, are not commonly available in natural shear zones and it is difficult to know what line was initially orthogonal to the boundary. The following sections describe some more practical methods.

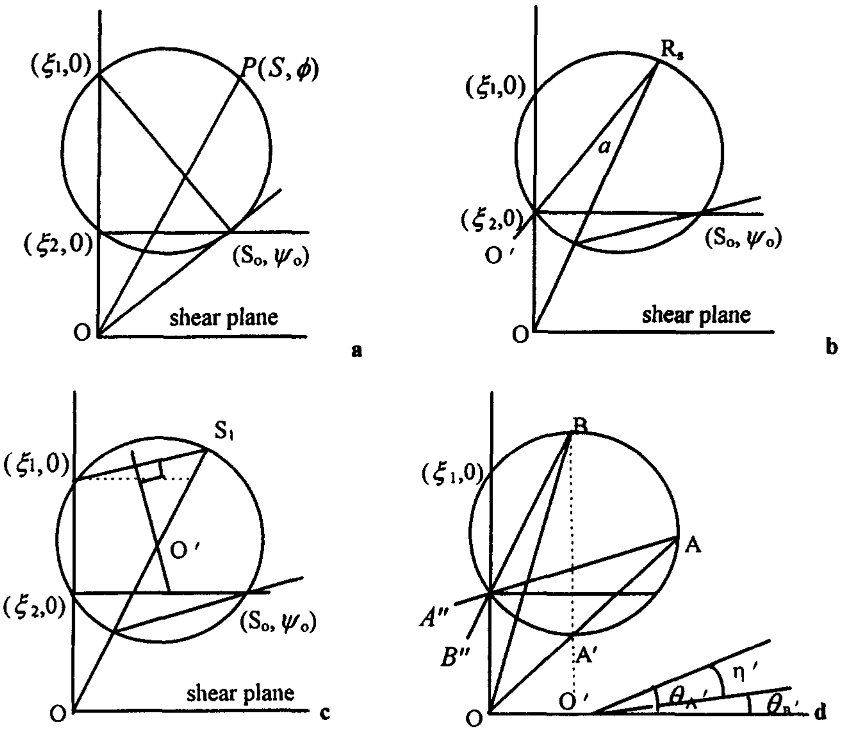

Fig. 2. Polar Mohr diagrams and their construction. (a) A polar Mohr diagram constructed with the stretches of two originally perpendicular strain markers, and the expression of stretch and angular shear (after Simpson and De Paor, 1993). Points (ξ_1, 0) and (ξ_2, 0) represent the deformation (ξ is the stretch) in the direction of the two eigenvectors, and point (ξ_1, 0) corresponds to the shear direction. Point (S_0, ψ_0) is the anchor point which represents the deformation of a marker originally normal to the shear plane. P (S, ϕ) presents the deformation in a given direction. S is the stretch and ϕ is the rotation with respect to the reference axis (the normal of the shear plane). (b), (c) and (d) Polar Mohr circles constructed with different methods (see text).

A polar Mohr diagram constructed with the measured stretch ratio, R_S*, and the angle* α *between the maximum principal stretch and the shear direction*

Using O as the origin (Fig. 2b), draw a line O–1–R_S (O–1 and O–R_S are equal to unit length and the stretch ratio R_S, respectively) and the Mohr circle with 1–R_S as a diameter. From the point R_S, draw a line R_S–O′ which makes the angle α with O–1–R_S. The intersection of the circle and line R_S–O′ is point (ξ_2, 0). The line joining the origin and point (ξ_2, 0) is the reference axis which is normal to the shear zone's boundary, and the other intersection with the circle is point (ξ_1, 0). The normal line of the axis at point (ξ_2, 0) intersects the circle at the anchor point (S_0, ψ_0). It should be noted that since no measure of area change is available, this circle is for a strain ellipse with a long axis R_S and a short axis of 1.

Construction of a polar Mohr diagram using the stretch ξ_1 *along the shear direction, maximum principal stretch* S_1 *and the angle* α *between* S_1 *and the shear direction*

Plot point (ξ_1, 0) on the reference axis and draw a line on one side from this point which makes an angle $90° - \alpha$ with the axis (Fig. 2c). Draw an arc of centre O and radius S_1 on the same side; the arc intersects the above line at S_1 making sure that O–S_1 = S_1 (the maximum stretch). The normal bisector of line (ξ_1, 0)–S_1 intersects O–S_1 at point O′. Using point O′ as the centre, draw a circle passing through points S_1 and (ξ_1, 0). This circle meets the reference axis at another point (ξ_2, 0). The normal line of the axis at this point intersects the circle at the anchor point (S_0, ψ_0).

A polar Mohr diagram constructed with the stretches S_A *and* S_B *in any two directions, the angle* η' *between them, and their angles* $\theta_{A'}$ *and* $\theta_{B'}$ *with the shear direction*

Two sheets of tracing paper and a net with a series of concentric circles are needed, e.g. a polar stereonet. On a tracing sheet, draw a line and take a segment OA = S_A. Draw line A–A″ through point A making angle OAA″ = $\theta_A{}'$ (Fig. 2d). Similarly, on the other sheet, first draw a line and take a segment OB = S_B, draw line B–B″ through point B making angle OBB″ = $\theta_B{}'$, and then, draw line B–O′ from point B making angle OBO′ = η' (Fig. 2d). Pin the two sheets at point O and place them on the net. Find a circle that passes points A, B and the intersection of A–A″ and B–B″, and make sure that the circle, lines B–O′ and O–A meet at a point A′ (Fig. 2d). The intersection of lines A–A″ and B–B″ is point (ξ_2, 0), and the line from point O passing through (ξ_2, 0) is the

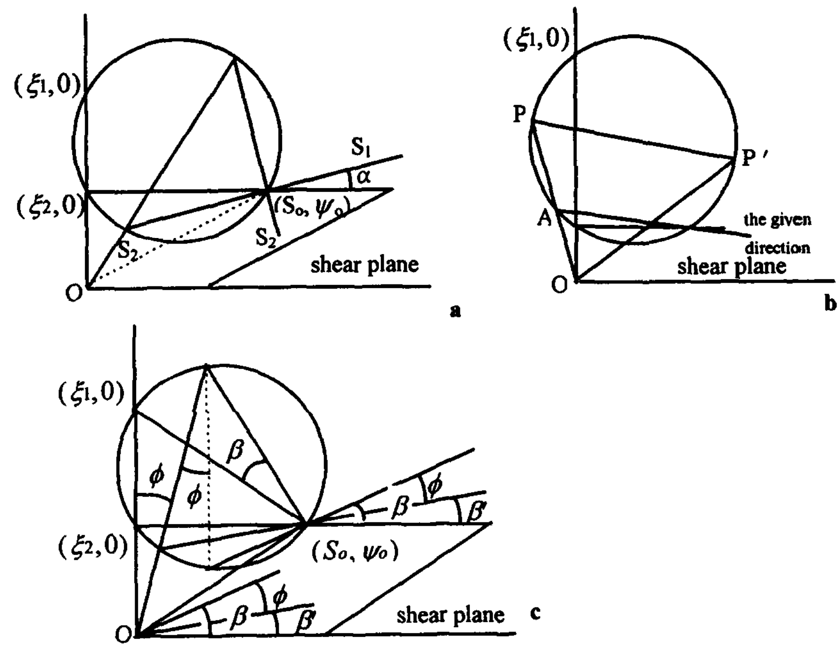

Fig. 3. Application of the polar Mohr diagram. Determining (a) the principal stretches and directions, and (b) the deformation (stretch and angular shear) in a given direction. (c) The angle β between a given line and shear direction before deformation and its rotation ϕ during deformation (after Simpson and De Paor, 1993).

reference axis. The anchor point and point $(\xi_1, 0)$ are obtained on the circle as stated above (Fig. 2d).

STRAIN ANALYSES WITH THE POLAR MOHR DIAGRAM FOR GENERAL SHEAR ZONES

With the polar Mohr diagrams constructed using the above methods, the principal stretches S_1 and S_2 (Fig. 3a), the original angle β of a given line (having a deformed angle β') with the shear direction and its rotation ϕ during deformation can be measured (Fig. 3c) (Simpson and De Paor, 1993). There are more measurements, in particular angle v, which can be made using the diagram.

The principal stretch direction

As in Fig. 3(a), the line passing through point S_2 and the anchor point represents the direction of maximum principal stretch axis. Between this line and the normal line of the reference axis is the angle α of the principal direction with the shear direction.

The stretch and angular shear in a given direction

Draw a line through the anchor point and parallel with the given direction (Fig. 3b). This line meets the circle at point A and another line from the origin, passing through point A, intersects the circle again at point P which represents the stretch and angular shear in the given direction. The length of O–P is the stretch in the given direction. Draw diameter P–P′, and lines O–P and O–P′. The angle POP′ is the angular shear ψ of the given direction.

Angle v and the kinematic vorticity number W_k *for a general shear zone*

Usually, a polar Mohr circle intersects the reference axis at two points $(\xi_1, 0)$ and $(\xi_2, 0)$ which represent the two eigenvectors of the shear zone. Point $(\xi_1, 0)$ equates with the shear direction. The line linking the anchor point and point $(\xi_2, 0)$ is normal to the reference axis. This line and diameter $(S_0, \psi_0)-(\xi_1, 0)$ make an angle which is the angle v between the two eigenvectors. W_k is computed with the angle v using the formula $W_k = \cos v$. If the centre of the circle falls on the reference axis and $v = 90°$, the case represents pure shear (Fig. 4c & d). If the circle is tangential with the reference axis and $v = 0°$, it corresponds to simple shear (Fig. 4e).

Analysis of changes in thickness

A shear zone has been thinned if $\xi_1 > \xi_2$, which means that it formed in an extensional setting (Fig. 4a). If $\xi_1 < \xi_2$, the shear zone has been thickened and formed in a compressive regime (Fig. 4b). The thickness is constant during simple shearing, when $\xi_1 = \xi_2$ (Fig. 4e). Suppose the present thickness of the shear zone is T_p, then its original thickness $T_o = T_p/\xi_2$.

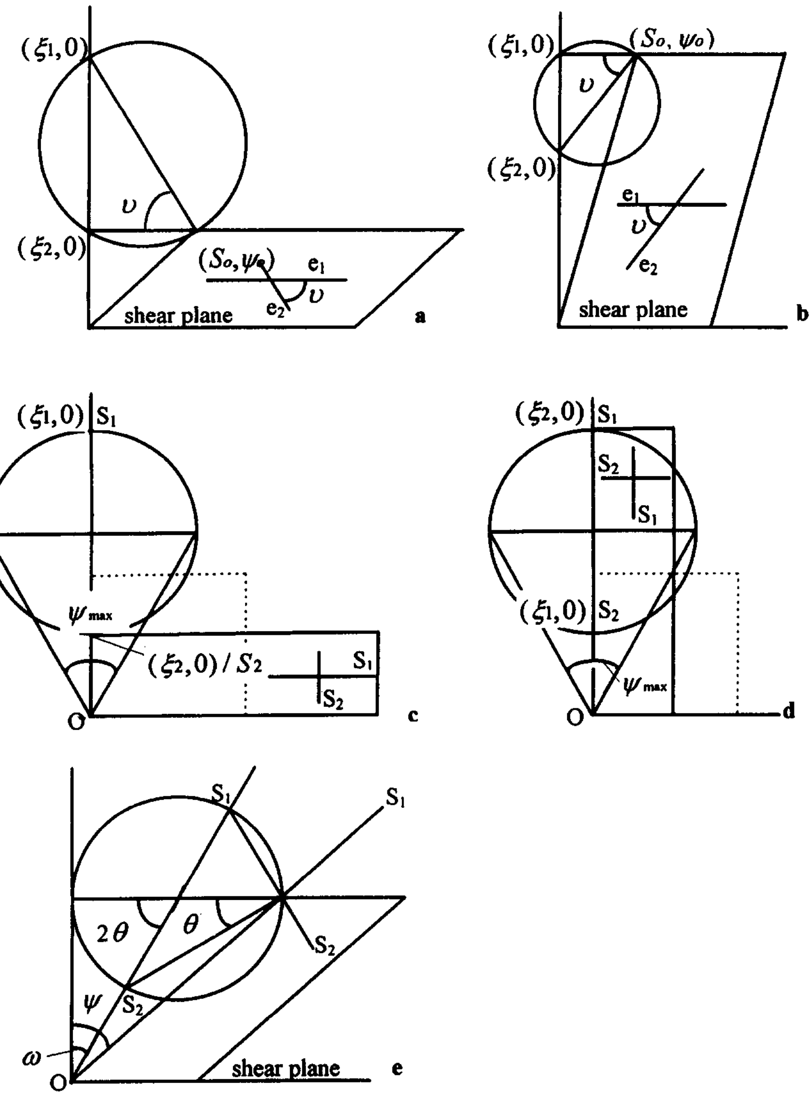

Fig. 4. Obtaining angle ν and an explanation of deformation history. (a) The case for a thinned general shear zone. (b) A thickened general shear zone (a and b are after Simpson and De Paor, 1993). (c) and (d) Special cases of coaxial deformation (c is after De Paor, 1987). (e) A polar Mohr circle representing simple shearing.

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